Published by Institute of Physics Publishing for SISSA

Received: *June 27, 2007*
Revised: *July 17, 2007*
Accepted: *August 4, 2007*
Published: *August 8, 2007*

$B \to \eta^{(\prime)}$ form factors in QCD

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Abstract: We calculate the semileptonic form factors $f_+^{B\to\eta}(q^2)$ and $f_+^{B\to\eta'}(q^2)$ from QCD sum rules on the light-cone (LCSRs), to NLO in QCD, and for small to moderate q^2, $0 \leq q^2 \leq 16\,\mathrm{GeV}^2$. We include in particular the so-called singlet contribution, i.e. weak annihilation of the B meson with the emission of two gluons which, thanks to the $\mathrm{U}(1)_\mathrm{A}$ anomaly, couple directly to $\eta^{(\prime)}$. This effect is included to leading-twist accuracy. This contribution has been neglected in previous calculations of the form factors from LCSRs. We find that the singlet contribution to $f_+^{B\to\eta'}$ can be up to 20%, while that to $f_+^{B\to\eta}$ is, as expected, much smaller and below 3%. We also suggest to measure the ratio $\mathcal{B}(B\to\eta' e\nu)/\mathcal{B}(B\to\eta e\nu)$ to better constrain the size of the singlet contribution.

Keywords: Weak Decays, B-Physics, Sum Rules, Phenomenological Models.

Contents

1. Introduction

$B \to \eta^{(\prime)}$ transitions are interesting for a number of reasons: at tree-level, they involve a $b \to u$ transition and hence are sensitive to the CKM matrix element $|V_{ub}|$. Its precise determination is crucial for the interpretation of the "tension" [1] that has emerged between the determination of $|V_{ub}|$ from, on the one hand, inclusive semileptonic $B \to X_u \ell\nu$ decays [2], and, on the other hand, global fits [1, 3] and the exclusive decay $B \to \pi\ell\nu$ [4–7]. The inclusive value of $|V_{ub}|$ is larger than that from other determinations and hints at a non-zero new-physics contribution to the B_d mixing phase ϕ_d, i.e. $\phi_d \neq 2\beta$ [8]. While an analysis of all available experimental and theoretical information on $B \to \pi\ell\nu$ found no "significant" disagreement between the exclusive and the inclusive values of $|V_{ub}|$ [6], the situation has changed very recently, when the HPQCD lattice collaboration reported a mistake in their calculation of the form factor $f_+^{B\to\pi}$ published in ref. [7]; the corrected form factor is larger and hence yields a smaller $|V_{ub}|$ [7]. The authors of ref. [6] have since then published an update [9] of their previous analysis and now conclude that the exclusive value of $|V_{ub}|$ is in perfect agreement with the determination from global fits and that "the hints of a disagreement with inclusive determinations of $|V_{ub}|$ are strengthened". Also very recently, Neubert has argued [10] that the value of $|V_{ub}|$ obtained by the HFAG collaboration [11] is dominated by observables with small efficiency and that, selecting observables with maximum efficiency instead, the resulting $|V_{ub}|$ is smaller than the HFAG average. Given this situation it is important to collect information on $|V_{ub}|$ also from other exclusive processes. $B \to \eta^{(\prime)}\ell\nu$ decays offer the opportunity for doing so.

Another reason why $B \to \eta^{(\prime)}$ transitions are interesting is their sensitivity to η-η' mixing and the effects of the $\mathrm{U(1)_A}$ anomaly, which is responsible for the large mass of the η' and also induces potentially large flavour-singlet contributions to amplitudes involving $\eta^{(\prime)}$. Indeed the unexpectedly large branching fractions of inclusive $B \to \eta' X$ and exclusive $B \to \eta' K$ decays, as compared to e.g. $B \to \pi$ transitions, have been attributed to an

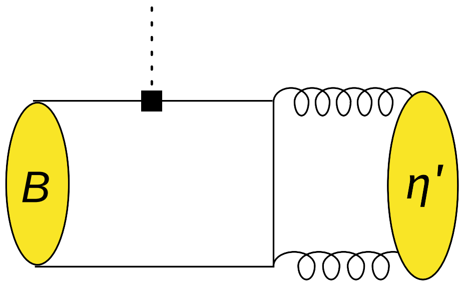

Figure 1: Flavour-singlet contribution to a generic $B \to \eta'$ transition.

enhanced flavour-singlet contribution [12], which is defined as the amplitude for producing either a quark-antiquark pair in a singlet state $(u\bar{u} + d\bar{d} + s\bar{s})$ which does not contain the B's spectator quark, or a pair of gluons, followed by hadronization into an $\eta^{(\prime)}$. A generic contribution of this type is shown in figure 1. In ref. [13] it was found that a rather large singlet-contribution of ca. 30% to the form factor $f_+^{B\to\eta'}$ would bring the central values of theoretical predictions for $B \to \eta' K$ observables in QCD factorisation into good agreement with experimental results, although the theoretical uncertainties are too large to allow a definite conclusion on the size of the singlet contributions. On the other hand, a more recent analysis of B decays with isosinglet final states, formulated in SCET, finds that, because of large experimental uncertainties of the data used to fit non-perturbative parameters, the singlet contribution to form factors is consistent with 0 [14].

While the interplay of singlet and octet contributions is well understood at the level of local matrix elements, i.e. decay constants (wave functions at the origin) [15 – 17], less is known about the shape of these wave functions, which are relevant for dynamical quantities like form factors. In frameworks based on QCD factorisation the mesons' Fock-state wave functions enter in the form of light-cone distribution amplitudes (DAs). Constraints on the leading parameters of these DAs have been obtained from the analysis of the $\eta^{(\prime)}\gamma$ transition form factor [18 – 20] and of the inclusive decay $Y(1S) \to \eta' X$ [20]. In principle, these DAs can also be constrained from a measurement of the form factors of $B \to \eta^{(\prime)}$, for instance from $\mathcal{B}(B \to \eta' \ell\nu)/\mathcal{B}(B \to \eta\ell\nu)$, as suggested in ref. [21].

Despite the strong phenomenological interest in the size of the singlet contribution to $f_+^{B\to\eta^{(\prime)}}$, there is, to the best of our knowledge, only a single calculation available, based on the perturbative QCD approach [22]. Ref. [22] finds that this contribution is negligible in $f_+^{B\to\eta}$, and reaches a few percent in $f_+^{B\to\eta'}$. Another well-known method for the calculation of $B \to$ light meson form factors are QCD sum rules on the light cone (LCSRs) [23 – 25]. Ref. [25], for instance, provides form factors for $B \to (\pi, K, \eta)$ decays, but does not include the singlet contribution to $B \to \eta$, nor a calculation of $B \to \eta'$ form factors. It is the purpose of this paper to remedy this situation and complete the calculation of $B \to$ light pseudoscalar meson form factors from LCSRs by including also the flavour-singlet contributions.

Our paper is organised as follows: in section 2 we define the two most common η-η' mixing schemes and review $\eta^{(\prime)}$ DAs. In section 3 we derive LCSRs for the $B \to \eta^{(\prime)}$ form

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factors. In section 4 we present results and conclude.

2. η and η' mixing and distribution amplitudes

There are two different mixing schemes in use to describe the η-η' system: the singlet-octet (SO) and the quark-flavour scheme (QF) [15]. In the former, the couplings of the relevant axial-vector currents to the meson $P = \eta, \eta'$ are given by

$$\langle 0|J^i_{\mu 5}|P(p)\rangle = i f^i_P p_\mu \quad (i = 1, 8), \tag{2.1}$$

where $J^8_{\mu 5}$ denotes the $\mathrm{SU(3)_F}$-octet and $J^1_{\mu 5}$ the $\mathrm{SU(3)_F}$-singlet axial-vector current, respectively. The four parameters f^i_P define two decay constants f_i of a hypothetical pure singlet or octet state $|\eta_i\rangle$ and also two mixing angles θ_i via

$$\begin{pmatrix} f^8_\eta & f^1_\eta \\ f^8_{\eta'} & f^1_{\eta'} \end{pmatrix} = \begin{pmatrix} \cos\theta_8 & -\sin\theta_1 \\ \sin\theta_8 & \cos\theta_1 \end{pmatrix} \begin{pmatrix} f_8 & 0 \\ 0 & f_1 \end{pmatrix}. \tag{2.2}$$

The advantage of this scheme is that the impact of the $\mathrm{U(1)_A}$ anomaly is plainly localised in f_1, via the divergence of the singlet current $J^1_{\mu 5}$, while $\theta_i \neq 0$ and $f_8 \neq f_\pi$ are $\mathrm{SU(3)_F}$-breaking effects. By the same token, the SO scheme also diagonalises the renormalisation-scale dependence of parameters and hence is very useful for checking the cancellation of divergences in perturbative calculations: f_8 and θ_i are scale-independent, while f_1 renormalises multiplicatively [26]:

$$\mu \frac{df_1}{d\mu} = -n_f \left(\frac{\alpha_s}{\pi}\right)^2 f_1 + O(\alpha_s^3). \tag{2.3}$$

In the QF mixing scheme, on the other hand, the basic axial-vector currents are

$$J^q_{\mu 5} = \frac{1}{\sqrt{2}} \left(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d\right), \qquad J^s_{\mu 5} = \bar{s}\gamma_\mu\gamma_5 s, \tag{2.4}$$

and the corresponding couplings to $P = \eta, \eta'$ are given by

$$\langle 0|J^r_{\mu 5}|P(p)\rangle = i f^r_P p_\mu \quad (r = q, s). \tag{2.5}$$

In complete correspondence to (2.2) one has

$$\begin{pmatrix} f^q_\eta & f^s_\eta \\ f^q_{\eta'} & f^s_{\eta'} \end{pmatrix} = \begin{pmatrix} \cos\phi_q & -\sin\phi_s \\ \sin\phi_q & \cos\phi_s \end{pmatrix} \begin{pmatrix} f_q & 0 \\ 0 & f_s \end{pmatrix}. \tag{2.6}$$

The basic difference to the SO scheme is that now the difference between the two angles $\phi_{q,s}$ is not caused by $\mathrm{SU(3)_F}$ effects, like that between θ_1 and θ_8, but by an OZI-rule violating contribution, as explained in ref. [16]. While the numerical values of θ_i differ largely, with typical values $\theta_8 \approx -20^\circ$ and $\theta_1 \approx -5^\circ$, one finds $\phi_s - \phi_q \lesssim 5^\circ$, with $\phi_q \approx \phi_s \approx 40^\circ$ [15, 16]. This led the authors of ref. [15] to suggest the QF scheme as an approximation to describe η-η' mixing, based on neglecting the difference $\phi_q - \phi_s$ (and all other OZI-breaking effects):

$$\phi \equiv \phi_{q,s}, \qquad \phi_q - \phi_s \equiv 0. \tag{2.7}$$

The advantage of this scheme is that it has only 3 parameters, f_q, f_s and ϕ, which implies that the mixing of states is the same as that of the decay constants:

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix}. \qquad (2.8)$$

The disadvantage is that, due to the neglection of OZI-breaking effects, the renormalisation-scale dependence of f_1 is not reproduced – as it is induced precisely by OZI-breaking terms [16]. While this is not really an issue numerically, as the scale-dependence of f_1 is a two-loop effect, eq. (2.3), the problem of the incompatibility of the QF scheme with the scale-dependence of parameters will come back at the level of non-local matrix elements, i.e. DAs, see below.

Given enough data to fix all independent parameters, there is no reason to prefer the QF over the SO scheme. For DAs, however, the SO scheme leads to a proliferation of unknown parameters, while the QF scheme is more restrictive, see below. For this reason we decide to use the QF scheme in this paper. Its basic parameters have been determined as [15]

$$f_q = (1.07 \pm 0.02) f_\pi, \qquad f_s = (1.34 \pm 0.06) f_\pi\,, \qquad \phi = 39.3^\circ \pm 1.0^\circ\,. \qquad (2.9)$$

This can be translated into values for the SO parameters as

$$\begin{aligned} f_8 &= \sqrt{\frac{1}{3}\, f_q^2 + \frac{2}{3} f_s^2} = (1.26 \pm 0.04) f_\pi\,, \\ f_1 &= \sqrt{\frac{2}{3}\, f_q^2 + \frac{1}{3} f_s^2} = (1.17 \pm 0.03) f_\pi\,, \\ \theta_8 &= \phi - \arctan[\sqrt{2} f_s/f_q] = -21.2^\circ \pm 1.6^\circ\,, \\ \theta_1 &= \phi - \arctan[\sqrt{2} f_q/f_s] = -9.2^\circ \pm 1.7^\circ\,. \end{aligned} \qquad (2.10)$$

Note that in the QF scheme $f_{q,s}$ are scale-independent parameters, and so is f_1 as obtained from the above relations. The SO decay constants can be expressed in terms of the QF ones and the angle ϕ as

$$\begin{pmatrix} f_\eta^8 & f_\eta^1 \\ f_{\eta'}^8 & f_{\eta'}^1 \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} f_q & 0 \\ 0 & f_s \end{pmatrix} \begin{pmatrix} \sqrt{\frac{1}{3}} & \sqrt{\frac{2}{3}} \\ -\sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} \end{pmatrix}. \qquad (2.11)$$

Let us now turn to light-cone DAs, that is the extension of matrix elements like (2.1) and (2.5) to those over non-local operators on the light-cone. This paper is not the place to give a thorough discussion of the properties of DAs, for which we refer to reviews [27] and to refs. [28, 29]. Suffice it to say that the DAs are ordered in terms of increasing twist, with the minimum, or leading, twist for meson DAs being two. Motivated by the structure of the evolution of DAs under a change of the renormalisation scale μ, they are expanded in terms of so-called asymptotic DAs multiplied by Gegenbauer polynomials. In the context of this paper it is important to recall that the $\mathrm{U}(1)_\mathrm{A}$ anomaly induces, in addition to two-quark DAs, also two-gluon DAs, of both leading and higher twist. Some properties of these higher-twist DAs have been studied in ref. [20]. In this paper we only include the effects of the leading-twist two-gluon DA, which is justified as its effects turn out to be small and higher-twist DAs are estimated to have even smaller impact. We will come back to that in section 4.

We define the twist-2 two-quark DAs of $\eta^{(\prime)}$ as [19]

$$\langle 0|\bar{\Psi}(z)\mathcal{C}_i \not{z}\gamma_5[z,-z]\Psi(-z)|P(p)\rangle = i(pz)f_P^i \int_0^1 du\, e^{i\xi(pz)}\phi_{2;P}^i(u)\,. \tag{2.12}$$

Here z_μ is a light-like vector, $z^2 = 0$, and $[x, y]$ stands for the path-ordered gauge factor along the straight line connecting the points x and y,

$$[x,y] = \mathrm{P}\exp\left[ig\int_0^1 dt\,(x-y)_\mu A^\mu(tx+(1-t)y)\right]. \tag{2.13}$$

u $(1-u)$ is the momentum fraction carried by the quark (antiquark) in the meson, ξ is short for $2u-1$. $\phi_{2;P}^i(u)$ is the twist-2 DA of the meson P with respect to the current whose flavour content is given by $\mathcal{C}_i$, with $\Psi = (u,d,s)$ the triplet of light-quark fields in flavour space. For the SO currents, one has $\mathcal{C}_1 = \mathbf{1}/\sqrt{3}$ and $\mathcal{C}_8 = \lambda_8/\sqrt{2}$, while for the QF currents $\mathcal{C}_q = (\sqrt{2}\mathcal{C}_1 + \mathcal{C}_8)/\sqrt{3}$ and $\mathcal{C}_s = (\mathcal{C}_1 - \sqrt{2}\mathcal{C}_8)/\sqrt{3}$, with λ_i the standard Gell-Mann matrices.

The gluonic twist-2 DA is defined as[1]

$$\langle 0|G_{\mu z}(z)[z,-z]\widetilde{G}^{\mu z}(-z)|P(p)\rangle = \frac{1}{2}\,(pz)^2\frac{C_F}{\sqrt{3}}f_P^1\int_0^1 du\, e^{i\xi(pz)}\psi_{2;P}^g(u)\,. \tag{2.14}$$

In order to perform the calculation of the correlation function defined in the next section, we also need the matrix element of the meson P over two gluon fields. Dropping the gauge factor $[z,-z]$, one has

$$\langle 0|A_\alpha^A(z)A_\beta^B(-z)|P(p)\rangle = \frac{1}{4}\,\epsilon_{\alpha\beta\rho\sigma}\,\frac{z^\rho p^\sigma}{(pz)}\,\frac{C_F}{\sqrt{3}}\,f_P^1\,\frac{\delta^{AB}}{8}\int_0^1 du\, e^{i\xi(pz)}\,\frac{\psi_{2;P}^g(u)}{u(1-u)}\,. \tag{2.15}$$

Because of the positive G-parity of η and η', the two-quark DAs are symmetric under $u \leftrightarrow 1-u$:

$$\phi_{2;P}^i(u) = \phi_{2;P}^i(1-u)\,; \tag{2.16}$$

they are expanded in terms of Gegenbauer polynomials as

$$\phi_{2;P}^i(u) = 6u(1-u)\left(1 + \sum_{n=2,4,\dots} a_n^{P,i}(\mu)C_n^{3/2}(\xi)\right) \quad (i = 1,8,q,s)\,; \tag{2.17}$$

$a_n^{P,i}$ are the quark Gegenbauer moments. As for the two-gluon DAs, the asymptotic DA is $u^{2j-1}(1-u)^{2j-1}$ with $j = 3/2$ the lowest conformal spin of the operator $G_{\mu z}$; the expansion goes in terms of Gegenbauer polynomials $C_n^{5/2}$. One can show that $\psi_{2;P}^g$ is antisymmetric:

$$\psi_{2;P}^g(u) = -\psi_{2:P}^g(1-u)\,; \tag{2.18}$$

[1]This definition refers to the "σ-rescaled" DA ϕ_g^σ in ref. [19] with $\sigma = \sqrt{3}/C_F$. It agrees with that used in refs. [20, 22], which means that we can use their results for the two-gluon Gegenbauer moment B_2^g without rescaling.

in particular $\int_0^1 du\, \psi^g_{2;P}(u) = 0$ and the local twist-2 matrix element $\langle 0|G_{\mu z}\widetilde{G}^{\mu z}|P\rangle$ vanishes. The non-vanishing coupling $\langle 0|G_{\alpha\beta}\widetilde{G}^{\alpha\beta}|P\rangle$ induced by the $\mathrm{U(1)_A}$ anomaly is a twist-4 effect. The corresponding matrix elements are given, in the QF scheme, by [15]:

$$\begin{aligned}\langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta_q\rangle &= f_s(m_\eta^2 - m_{\eta'}^2)\sin\phi\cos\phi\,,\\ \langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta_s\rangle &= f_q(m_\eta^2 - m_{\eta'}^2)/\sqrt{2}\sin\phi\cos\phi\,.\end{aligned} \tag{2.19}$$

We will estimate the size of these effects in section 4. There are no twist-3 two-gluon DAs and the remaining twist-4 DAs also have vanising normalisation, see ref. [20]. The conformal expansion of the twist-2 two-gluon DA reads

$$\psi^g_{2;P}(u,\mu) = u^2(1-u)^2 \sum_{n=2,4,\dots} B_n^{P,g}(\mu) C_{n-1}^{5/2}(\xi) \tag{2.20}$$

with the gluonic Gegenbauer moments $B_n^{P,g}$. In this paper, we truncate both $\phi^i_{2;P}$ and $\psi^g_{2;P}$ at $n=2$. This is due to the fact that our knowledge about these higher-order Gegenbauer moments is very restricted. An estimate of the effect of higher Gegenbauer moments in $\phi_{2;\pi}$ on the $B\to\pi$ form factor f_+^π has been given in ref. [30], based on a certain class of models for the full DA beyond conformal expansion. The effect of neglecting $a^\pi_{n\geq 4}$ we found to be very small, $\sim 2\%$. We expect the truncation error from neglecing $B^g_{n\geq 4}$ to be of similar size.

$\phi^1_{2;P}$ and $\psi^g_{2;P}$ mix upon evolution in μ, see for instance ref. [19]. This amounts to a mixing of $a_2^{P,1}$ and $B_2^{P,g}$, resulting in the renormalisation-group equation, to LO accuracy,

$$\mu\frac{d}{d\mu}\begin{pmatrix} a_2^1 \\ B_2^g\end{pmatrix} = -\frac{\alpha_s}{4\pi}\begin{pmatrix} \frac{100}{9} & -\frac{10}{81} \\ -36 & 22\end{pmatrix}\begin{pmatrix} a_2^1 \\ B_2^g\end{pmatrix}, \tag{2.21}$$

where for simplicity we have dropped the superscript P. We only quote the solution for a_2^1:

$$\begin{aligned}a_2^1(\mu) = {}& \left[\left(\frac{1}{2} - \frac{49}{2\sqrt{2761}}\right) L^{\gamma_2^+/(2\beta_0)} + \left(\frac{1}{2} + \frac{49}{2\sqrt{2761}}\right) L^{\gamma_2^-/(2\beta_0)}\right] a_2^1(\mu_0)\\ &+ \frac{5}{9\sqrt{2761}}\left[L^{\gamma_2^-/(2\beta_0)} - L^{\gamma_2^+/(2\beta_0)}\right] B_2^g(\mu_0)\end{aligned} \tag{2.22}$$

with $L = \alpha_s(\mu)/\alpha_s(\mu_0)$ and the anomalous dimensions $\gamma_2^\pm = (149 \pm \sqrt{2761})/9$. This is to be compared to the evolution of the octet Gegenbauer moment:

$$a_2^8(\mu) = L^{50/(9\beta_0)} a_2^8(\mu_0)\,. \tag{2.23}$$

Numerically, the evolution of a_2^1 does not differ much from that of a_2^8, for a wide range of B_2^g: assume $a_2^8(1\,\mathrm{GeV}) \equiv a_2^1(1\,\mathrm{GeV})$, as is the case for a strict imposition of the QF scheme. Choose $a_2^8(1\,\mathrm{GeV}) = 0.2$, as indicated by our knowledge of twist-2 DAs of the π; then we have $a_2^8(2.4\,\mathrm{GeV}) = 0.137$ from (2.23); 2.4 GeV is a typical scale in the calculation of form factors from LCSRs. In figure 2 we show the results of the evolution of the

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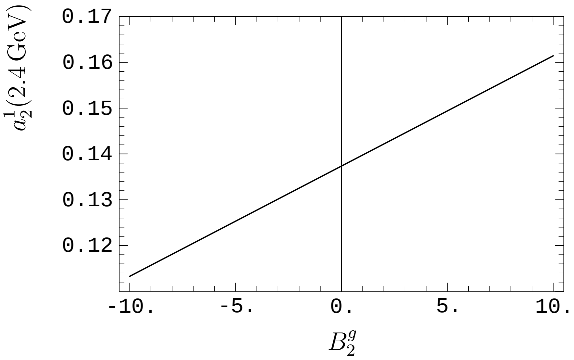

Figure 2: Dependence of $a_2^1(2.4\,\text{GeV})$ on $B_2^g(1\,\text{GeV})$, eq. (2.22), for $a_2^1(1\,\text{GeV}) = 0.2$.

singlet Gegenbauer moment a_2^1 from 1 to 2.4 GeV, from eq. (2.22), for the range of gluon Gegenbauer moments $|B_2^g(1\,\text{GeV})| < 10$. Evidently the impact of the different anomalous dimensions of a_2^1 and a_2^8 is negligible ($a_2^1(2.4\,\text{GeV}) = 0.137$ for $B_2^g = 0$) and the mixing of B_2^g into a_2^1 is smaller than 20% within the range of B_2^g considered.

At this point we would like to come back to the impact of evolution on the consistency of the QF scheme. We introduce the twist-2 two-quark DAs ϕ_2^i, $i = 1, 8, q, s$, corresponding to the basis states $|\eta_i\rangle$ in the SO and QF scheme, respectively. We then have, in terms of the quark valence Fock states $|q\bar{q}\rangle$ and $|s\bar{s}\rangle$ [19]:

$$|\eta_q\rangle \sim \phi_2^q(u)|q\bar{q}\rangle + \phi_2^{\text{OZI}}(u)|s\bar{s}\rangle\,, \quad |\eta_s\rangle \sim \phi_2^{\text{OZI}}(u)|q\bar{q}\rangle + \phi_2^s(u)|s\bar{s}\rangle\,, \tag{2.24}$$

where $q\bar{q}$ is shorthand for $(u\bar{u} + d\bar{d})/\sqrt{2}$ and

$$\phi_2^q = \frac{1}{3}\,(\phi_2^8 + 2\phi_2^1)\,, \quad \phi_2^s = \frac{1}{3}\,(2\phi_2^8 + \phi_2^1)\,, \quad \phi_2^{\text{OZI}} = \frac{\sqrt{2}}{3}(\phi_2^1 - \phi_2^8)\,. \tag{2.25}$$

In the QF scheme, the "wrong-flavour" DA ϕ_2^{OZI}, which is generated by OZI-violating interactions, is set to 0. Once this is done at a certain scale, however, the different evolution of a_n^1 and a_n^8, eqs. (2.22) and (2.23), will generate a non-zero ϕ_2^{OZI} already to LO accuracy. A consistent implementation of the QF scheme hence requires one to either set $a_n^{1,8} \equiv 0$ and also $B_n^g \equiv 0$, or to set $a_n^8 \equiv a_n^1$ and neglect the different scale-dependence of these parameters. In practice, however, the QF scheme is an approximation anyway, motivated by the observed smallness of one parameter, the difference of mixing angles $\phi_s - \phi_q$. The induced non-zero DA ϕ_2^{OZI} is numerically very small for the scales relevant for our calculation, $\mu = 1\,\text{GeV}$ and 2.4 GeV. We hence implement the QF scheme for DAs as follows: we set $\phi_2^1 \equiv \phi_2^8$ at the scale $\mu = 1\,\text{GeV}$, which, by virtue of (2.25), implies $\phi_2^q \equiv \phi_2^s$ at the same scale. We then evolve a_2 according to the scaling-law for the octet Gegenbauer moment, eq. (2.23).[2] We also set $\psi_{2;\eta}^g = \psi_{2;\eta'}^g$; again any SU(3)$_\text{F}$ breaking of this relation is expected to have only very small impact on $f_+^{B\to\eta^{(\prime)}}$. The twist-2 parameters used in our calculation are then reduced to 2: a_2 and B_2^g. For error estimates, we will also sometimes distinguish between a_2^η and $a_2^{\eta'}$.

[2]This is equivalent to imposing the QF-scheme relation $a_2^1 = a_2^8$ as the scale $\mu = 2.4\,\text{GeV}$ and defining B_2^g as $B_2^g(2.4\,\text{GeV})$.

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As far as numerics is concerned, we assume that the bulk of $\mathrm{SU(3)_F}$-breaking effects is described by the decay constants via $f_q \neq f_\pi$, and that $\mathrm{SU(3)_F}$ breaking in Gegenbauer moments is subleading. This motivates setting $a_2^q = a_2^\pi$, with $a_2^\pi(1\,\mathrm{GeV}) = 0.25 \pm 0.15$ as an average over a large number of calculations and fits to experimental data [29]; this number also agrees with a recent lattice determination [31]. $a_2^q = a_2^\pi$ is justified as, as discussed in ref. [29], there is no evidence for noticeable SU(3)-breaking effects between a_2^π and a_2^K and the main SU(3)-breaking in the DAs is due to non-zero odd Gegenbauer moments. In this work we only need a_2^q, and as a QCD sum rule for this parameter would look essentially the same as that for a_2^π, except for a slightly different value for the decay constant, $f_\pi \neq f_q$, and different numerical values for the continuum threshold s_0 and the window in the Borel parameter M^2, we see no plausible source for large SU(3) breaking between a_2^π and a_2^q. To the best of our knowledge, no calculation of B_2^g is available. Results from fits to data have been obtained from the $\eta^{(\prime)}\gamma$ transition form factor, yielding $B_2^g(1\,\mathrm{GeV}) = 9 \pm 12$ [19], and the combined analysis of this form factor and the inclusive decay $Y(1S) \to \eta' X$ yielding $B_2^g(1.4\,\mathrm{GeV}) = 4.6 \pm 2.5$ [20]. These results, however, have to be taken cum grano salis as they are highly correlated with the simultaneous determination of a_2^1 and a_2^8 from the same data, yielding $a_2^1(1\,\mathrm{GeV}) = -0.08 \pm 0.04$, $a_2^8(1\,\mathrm{GeV}) = -0.04 \pm 0.04$ [19] and $a_2^1(1.4\,\mathrm{GeV}) = a_2^8(1.4\,\mathrm{GeV}) = -0.054 \pm 0.029$ [20]. The same analysis applied to the $\pi\gamma$ form factor returns $a_2^\pi(1\,\mathrm{GeV}) = -0.06 \pm 0.03$ [32]. These results are not really compatible with those from the direct calculation of a_2^π from lattice and QCD sum rules; in particular the sign of a_2^π is unambiguously fixed as being positive. A possible reason for this discrepancy is the neglect of higher-order terms in the light-cone expansion and that, in addition, as one of the photons in the process is nearly real with virtuality $q^2 \approx 0$, one also has to take into account long-distance photon interactions, of order $1/\sqrt{q^2}$ [33]. For this reason, we assume the very conservative range $B_2^g(2.4\,\mathrm{GeV}) = 0 \pm 20$ in the remainder of this paper.

As far as higher-twist DAs are concerned, we only need those involving currents with flavour content $\bar{q}q = (\bar{u}u + \bar{d}d)/\sqrt{2}$. In line with the implementation of the QF scheme for twist-2 DAs, we include $\mathrm{SU(3)_F}$ breaking only via the decay constants and set

$$\frac{1}{f^q_{\eta^{(\prime)}}}\,\langle 0|\bar{\Psi}(z)C_q[z,-z]\Gamma\Psi(-z)|\eta^{(\prime)}(p)\rangle = \frac{1}{f_\pi}\,\langle 0|\bar{d}(z)[z,-z]\Gamma u(-z)|\pi^-(p)\rangle\,,$$

$$\frac{1}{f^q_{\eta^{(\prime)}}}\,\langle 0|\bar{\Psi}(z)[z,vz]G(vz)C_q\Gamma[vz,-z]\Psi(-z)|\eta^{(\prime)}(p)\rangle = \frac{1}{f_\pi}\,\langle 0|\bar{d}(z)[z,vz]G(vz)\Gamma[vz,-z]u(-z)|\pi^-(p)\rangle\,, \tag{2.26}$$

where Γ is the relevant Dirac structure and $G(vz)$ the gluon field-strength tensor. The precise definitions of all twist-3 and 4 DAs, as well as up-to-date numerical values of the π's hadronic parameters can be found in ref. [29]. Let us shortly comment on the validity of this treatment for twist-3 two-quark DAs. As is well known, the normalisation of these DAs is given, for the π, by $f_\pi m_\pi^2/(2m_q)$ and enters the light-cone sum rules for $B \to \pi$ transitions as a $1/m_b$ correction, see explicit formulas for the corresponding D form factor in ref. [34]. Although suppressed by one power of the heavy quark mass, this

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contribution is numerically non-negligible due to the chiral enhancement factor. Following the above implementation of SU(3) breaking, we set $f_\pi m_\pi^2/(2m_q) \to f_q m_\pi^2/(2m_q)$ for η_q (the corresponding quantity for η_s is not needed). In contrast, the inclusion of all SU(3) effects leads one to consider the quantity

$$h_q = f_q(m_\eta^2 \cos^2\phi + m_{\eta'}^2 \sin^2\phi) - \sqrt{2} f_s(m_{\eta'}^2 - m_\eta^2)\sin\phi\cos\phi\,; \tag{2.27}$$

the normalisation of the twist-3 DAs of η_q is given by $h_q/(2m_q)$. To leading order in the chiral expansion and $1/N_c$ expansion, $h_q \to f_q m_\pi^2 = 0.0025\,\mathrm{GeV}^3$, which is the value used in our scheme. As discussed in ref. [13], the full expression (2.27) yields $h_q = (0.0015 \pm 0.004)\,\mathrm{GeV}^3$, i.e. a 200% uncertainty, if the errors of $f_{q,s}$ and ϕ are treated as uncorrelated. The large error is due to a cancellation between the two terms in (2.27). As the parameter we need is actually $h_q/(2m_q)$, with m_q not very well constrained (yet) from lattice calculations[3] and the correlation of the errors of $f_{q,s}$ and ϕ is not known, we feel that a total 250% uncertainty of $h_q/(2m_q)$ is slightly exaggerated and an artifact of the numerical cancellation. Instead, we work to leading order in the chiral expansion and set $h_q/(2m_q) = f_q B_0$, with $B_0 = m_\pi^2/(2m_q) = -2\langle 0|\bar{q}q|0\rangle/f_\pi^2$ [28]. $\langle 0|\bar{q}q|0\rangle$, the quark condensate, is the order parameter of chiral symmetry breaking and known from QCD sum rules to have the value $\langle 0|\bar{q}q|0\rangle = (-0.24 \pm 0.01)^3\,\mathrm{GeV}^3$. From this, one finds $B_0 = (1.6 \pm 0.2)\,\mathrm{GeV}$ [28], which, together with the error on f_q, implies a total 15% uncertainty for the normalisation of the twist-3 DAs. This is the standard treatment of these terms in the framework of light-cone sum rules.

3. LCSRs for gluonic contributions

The key idea of light-cone sum rules is to consider a correlation function of the weak current and a current with the quantum numbers of the B meson, sandwiched between the vacuum and an η or η' state. For large (negative) virtualities of these currents, the correlation function is, in coordinate-space, dominated by distances close to the light-cone and can be discussed in the framework of light-cone expansion. In contrast to the short-distance expansion employed by conventional QCD sum rules à la SVZ [36], where non-perturbative effects are encoded in vacuum expectation values of local operators with vacuum quantum numbers, the condensates, LCSRs rely on the factorisation of the underlying correlation function into genuinely non-perturbative and universal hadron DAs ϕ. The DAs are convoluted with process-dependent amplitudes T_H, which are the analogues of the Wilson coefficients in the short-distance expansion and can be calculated in perturbation theory. Schematically, one has

$$\text{correlation function} \sim \sum_n T_H^{(n)} \otimes \phi_n. \tag{3.1}$$

The expansion is ordered in terms of contributions of increasing twist n. The light-cone expansion is matched to the description of the correlation function in terms of hadrons by

[3] A recent unquenched calculation yields $\overline{m} \equiv (m_u+m_d)/2 = (3.54^{+0.64}_{-0.35})$ MeV at the scale $\mu = 2$ GeV [35].

analytic continuation into the physical regime and the application of a Borel transformation, which introduces the Borel parameter M^2 and exponentially suppresses contributions from higher-mass states. In order to extract the contribution of the B meson, one describes the contribution of other hadron states by a continuum model, which introduces a second model parameter, the continuum threshold s_0. The sum rule then yields the form factor in question, f_+, multiplied by the coupling of the B meson to its interpolating field, i.e. the B meson's leptonic decay constant f_B.

LCSRs are available for the $B \to \pi, K$ form factor f_+ to $O(\alpha_s)$ accuracy for the twist-2 and part of the twist-3 contributions and at tree-level for higher-twist (3 and 4) contributions [25].

We define the $B \to P$ form factors as

$$\langle P(p)|\bar{u}\gamma_\mu b|B(p+q)\rangle = \left\{(2p+q)_\mu - \frac{m_B^2 - m_P^2}{q^2}\, q_\mu\right\} \frac{f_+^P(q^2)}{\sqrt{2}} + \frac{m_B^2 - m_P^2}{q^2}\, q_\mu\, \frac{f_0^P(q^2)}{\sqrt{2}}\,. \quad (3.2)$$

Note that we include a factor $1/\sqrt{2}$ on the right-hand side. This is to ensure that in the limit of $\mathrm{SU(3)_F}$ symmetry and no η-η' mixing $f_+^\eta = f_+^\pi$.

In the semileptonic decay $B \to \eta^{(\prime)} l\nu_l$ the form factor f_0^P $(P = \eta, \eta')$ enters proportional to the lepton mass m_l^2 and hence is irrelevant for light leptons $(l = e, \mu)$, where only f_+^P matters. The semileptonic decay can be used to determine the size of the CKM matrix element $|V_{ub}|$ from the spectrum

$$\frac{d\Gamma}{dq^2}(B \to \eta^{(\prime)} l\nu_l) = \frac{G_F^2 |V_{ub}|^2}{192\pi^3 m_B^3} \lambda^{3/2}(q^2) |f_+^P(q^2)|^2\,, \quad (3.3)$$

where $\lambda(x) = (m_B^2 + m_P^2 - x)^2 - 4m_B^2 m_P^2$. Alternatively, as we shall see, the ratio of branching ratios $\mathcal{B}(B \to \eta'\ell\nu)/\mathcal{B}(B \to \eta\ell\nu)$ can be used to constrain the gluonic Gegenbauer moment B_2^g.

Our starting point for calculating f_+^P is the correlation function

$$\begin{aligned}\Pi_\mu^P(p,q) &= i\int d^4x\, e^{i(qx)} \langle P(p)|T[\bar{u}\gamma_\mu b](x) j_B^\dagger(0)|0\rangle \\ &= \Pi_+^P(q^2, p_B^2)(2p+q)_\mu + \ldots\end{aligned} \quad (3.4)$$

where $j_B = m_b \bar{u} i\gamma_5 b$ is the interpolating field for the B meson and $p_B^2 = (p+q)^2$ its virtuality. For

$$m_b^2 - p_B^2 \geq O(\Lambda_{\mathrm{QCD}} m_b), \qquad m_b^2 - q^2 \geq O(\Lambda_{\mathrm{QCD}} m_b), \quad (3.5)$$

the correlation function (3.4) is dominated by light-like distances and therefore accessible to an expansion around the light-cone. The above conditions can be understood by demanding that the exponential factor in (3.4) vary only slowly. The light-cone expansion is performed by integrating out the transverse and "minus" degrees of freedom and leaving only the longitudinal momenta of the partons as relevant degrees of freedom. The integration over transverse momenta is done up to a cutoff, μ_{IR}, all momenta below which are included in a the DAs ϕ_n. Larger transverse momenta are calculated in perturbation theory. The

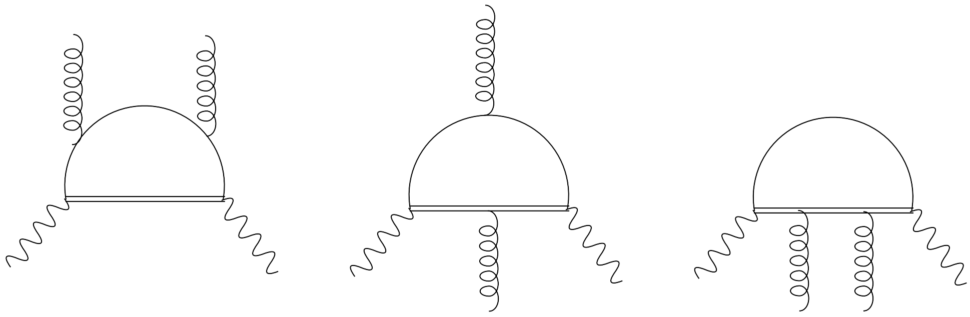

Figure 3: Feynman diagrams of the gluonic contributions. The double line denotes the b quark, the photon-like lines the currents in the correlation function Π_μ^P. The first diagram is divergent, the other two are convergent.

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correlation function is hence decomposed, or factorised, into perturbative contributions T and nonperturbative contributions ϕ, which both depend on the longitudinal parton momenta and the factorisation scale $\mu_{\rm IR}$. The schematic relation (3.1) can then be written in more explicit form, including only two-particle DAs, as

$$\Pi_+^P(q^2, p_B^2) = \sum_n \int_0^1 du\, T^{(n)}(u, q^2, p_B^2, \mu_{\rm IR})\phi_{n;P}(u, \mu_{\rm IR}). \tag{3.6}$$

As Π_+ itself is independent of the arbitrary scale $\mu_{\rm IR}$, the scale-dependence of $T^{(n)}$ and ϕ_n must cancel each other. If there is more than one contribution of a given twist, they will mix under a change of $\mu_{\rm IR}$ and it is only in the sum of all such contributions that the residual $\mu_{\rm IR}$ dependence cancels. This is what happens with the two-quark and two-gluon contributions to $B \to \eta^{(\prime)}$. Eq. (3.6) is called a "collinear" factorisation formula, as the momenta of the partons in P are collinear with the P's momentum. Its validity actually has to be verified, which is done precisely by checking that the $\mu_{\rm IR}$ dependence cancels. In ref. [25] it has been shown that the above formula holds to $O(\alpha_s)$ accuracy for two-quark twist-2 and -3 contributions.

In calculating the correlation function, we use relation (2.8) between $|\eta^{(\prime)}\rangle$ and the QF basis states $|\eta_{q,s}\rangle$, so that

$$\Pi_\mu^\eta = \frac{1}{\sqrt{2}}\left(\Pi_\mu^q \cos\phi - \Pi_\mu^s \sin\phi\right), \quad \Pi_\mu^{\eta'} = \frac{1}{\sqrt{2}}\left(\Pi_\mu^q \sin\phi + \Pi_\mu^s \cos\phi\right). \tag{3.7}$$

As the correlation function involves the current $\bar{u}\gamma_\mu b$, Π_μ^s vanishes to leading order in α_s and at $O(\alpha_s)$ is due only to gluonic Fock states of the meson. Π_μ^q, on the other hand, receives contributions from both quark and gluon states. The quark contributions have been calculated in ref. [25] for $B \to \pi$, including $O(\alpha_s)$ corrections to twist-2 and -3 contributions, and to tree-level accuracy for twist-4 contributions. The corresponding expressions yield Π_+^q, with the replacement $f_\pi \to f_q$.

In order to obtain the singlet contribution to Π_+^P, one needs to calculate the diagrams shown in figure 3. The projection of the gluon fields onto the DA $\psi_{2;P}^g$ can be read off eq. (2.15). The explicit formula is given in the appendix. We check the result by verifying the cancellation of the $\mu_{\rm IR}$-dependent terms as described above. The relevant term in the quark Gegenbauer moment a_2 is

$$\Pi_+^q \sim 18 f_q F(p_B^2, q^2) a_2 \left(1 + \frac{\alpha_s}{4\pi}\,\frac{50}{9}\,\ln\frac{\mu_{\rm IR}^2}{m_b^2}\right), \tag{3.8}$$

where $F(p_B^2, q^2)$ is a function of p_B^2 and q^2. The logarithmic terms in the convolution of the gluonic diagrams of figure 3 with $\psi^g_{2;P}$ read

$$\Pi_+^P \sim -\frac{10}{9\sqrt{3}}\,\frac{\alpha_s}{4\pi}\,B_2^g f_P^1 \ln\frac{\mu_{\rm IR}^2}{m_b^2}\,F(p_B^2, q^2)\,. \tag{3.9}$$

One can easily convince oneself by expressing f_q via eq. (2.11) in terms of f_η^1 and $f_{\eta'}^1$, respectively, and inserting (3.8) into (3.7), that the renormalisation-group equation (2.21) is fulfilled.

The final LCSR for f_+^P then reads

$$e^{-m_B^2/M^2}\,m_B^2 f_B\,\frac{f_+^P(q^2)}{\sqrt{2}} = \int_{m_b^2}^{s_0} ds\, e^{-s/M^2}\,\frac{1}{\pi}\,{\rm Im}\,\Pi_+^P(s, q^2)\,, \tag{3.10}$$

with the sum-rule specific parameters M^2, the Borel parameter, and s_0, the continuum threshold.

4. Results and discussion

Let us now give the results for the form factors. As usual, we replace f_B in the sum rule (3.10) by its QCD sum rule to $O(\alpha_s)$ accuracy; this reduces the dependence of the results on $m_b = (4.80\pm0.05)\,$GeV. In figure 4 we plot $f_+^\eta(0)$ and $f_+^{\eta'}(0)$, respectively, as functions of the Borel parameter M^2. The continuum threshold is chosen as $s_0 = 34.2\,{\rm GeV}^2$, which corresponds to the optimum s_0 for the sum rule for f_B [25]. The factorisation scale $\mu_{\rm IR}$ is chosen as intermediate between m_b and an intrinsic hadronic scale 1 GeV; following our earlier papers, we choose $\mu_{\rm IR}^2 = m_B^2 - m_b^2$. The dependence of $f_+^{\eta,\eta'}$ on M^2 is small in the Borel-window $M^2 > 6\,{\rm GeV}^2$. We estimate the uncertainty in M^2 as the variation of the form factor in the interval $M^2 \in [6, 14]\,{\rm GeV}^2$. In figure 4, we also show the dependence of the form factors on s_0 by varying it by $\pm0.7\,{\rm GeV}^2$; also this dependence is rather small. The central values of the most relevant hadronic input parameters are $m_b = 4.8\,$GeV, $a_2^{\eta,\eta'}(1\,{\rm GeV}) = 0.25$ and $B_2^g = 0$. As expected, $f_+^\eta(0)$ is not very sensitive to the singlet contribution parameter B_2^g (red/dashed-dotted curves), but rather sensitive to the Gegenbauer moment a_2 (green/short-dashed curves). For $f_+^{\eta'}(0)$, on the other hand, the dependence on B_2^g is more pronounced than that of a_2. Varying all relevant parameters within their respective ranges, i.e. $\Delta m_b = \pm0.05\,$GeV, $\Delta a_2(1\,{\rm GeV}) = \pm0.15$ and $\Delta B_2^g = \pm20$, as well

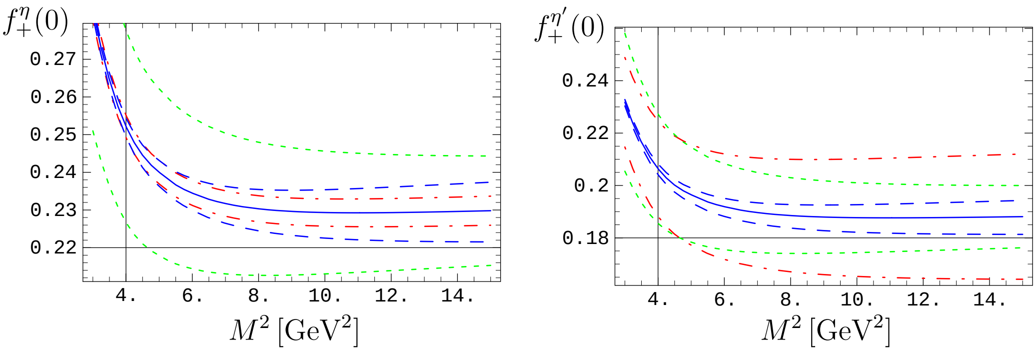

Figure 4: [Colour online] $f_+^\eta(0)$ (left) and $f_+^{\eta'}(0)$ (right) as a function of the Borel parameter M^2 and various choices of input parameters. Solid curves: central values of input parameters and $s_0 = 34.2\,\mathrm{GeV}^2$. Long-dashed (blue) curves: s_0 varied by $\pm 0.7\,\mathrm{GeV}^2$. Short-dashed (green) curves: $a_2(1\,\mathrm{GeV})$ varied by ± 0.15. Dash-dotted (red) curves: B_2^g varied by ± 10.

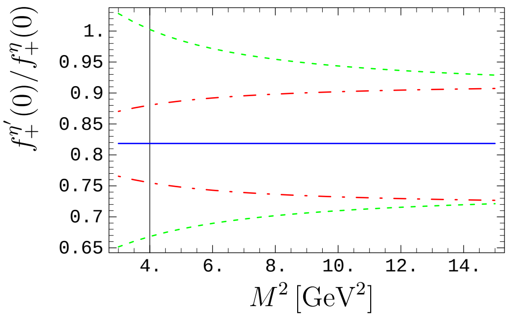

Figure 5: [Colour online] $f_+^{\eta'}(0)/f_+^\eta(0)$ as a function of the Borel parameter M^2 and various choices of input parameters. Solid (blue) line: central values of input parameters, which corresponds to $f_+^{\eta'}(0)/f_+^\eta(0) \equiv \tan\phi = 0.814$. Dash-dotted (red) curves: B_2^g varied by ± 10. Short-dashed (green) curves: $a_2^{\eta,\eta'}(1\,\mathrm{GeV})$ varied independently: $a_2^\eta = 0.1$, $a_2^{\eta'} = 0.4$ and $a_2^{\eta'} = 0.4$, $a_2^\eta = 0.1$.

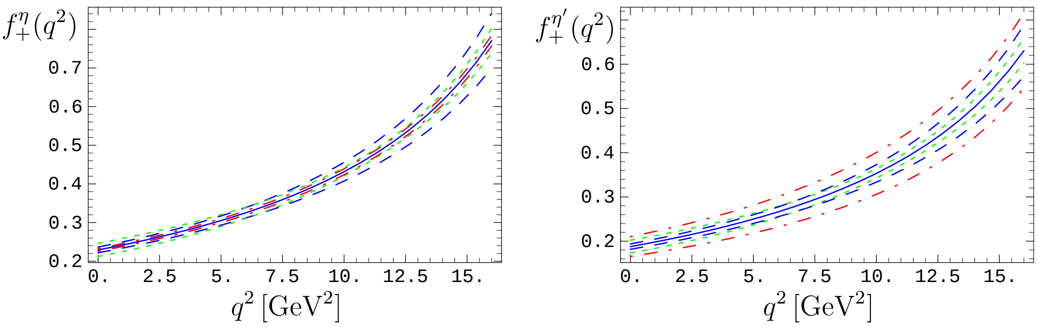

Figure 6: [Colour online] $f_+^\eta(q^2)$ (left) and $f_+^{\eta'}(q^2)$ (right) as a function of the momentum transfer q^2 and various choices of input parameters. Solid curves: central values of input parameters and $M^2 = 10\,\mathrm{GeV}^2$, $s_0 = 34.2\,\mathrm{GeV}^2$. Long-dashed (blue) curves: s_0 varied by $\pm 0.7\,\mathrm{GeV}^2$ and M^2 by $\pm 4\,\mathrm{GeV}^2$. Short-dashed (green) curves: $a_2(1\,\mathrm{GeV})$ varied by ± 0.15, f_q/f_π by ± 0.02 and ϕ by $\pm 1°$. Dash-dotted (red) curves: B_2^g varied by ± 10.

as all twist-3 and twist-4 parameters within the ranges given in ref. [29], we find

$$\begin{aligned}
f_+^\eta(0) &= 0.229 \pm 0.005(M^2) \pm 0.006(s_0) \pm 0.016(a_2^\eta) \pm 0.007(B_2^g) \pm 0.005(f_q,\phi) \\
&\quad \pm 0.011(\mathrm{T3}) \pm 0.001(\mathrm{T4}) \pm 0.007(f_B, m_b) \\
&= 0.229 \pm 0.024(\mathrm{param.}) \pm 0.011(\mathrm{syst.})\,, \qquad (4.1) \\
f_+^{\eta'}(0) &= 0.188 \pm 0.004(M^2) \pm 0.005(s_0) \pm 0.013(a_2^{\eta'}) \pm 0.043(B_2^g) \pm 0.005(f_q,\phi) \\
&\quad \pm 0.009(\mathrm{T3}) \pm 0.005(\mathrm{T4}) \pm 0.006(f_B, m_b) \\
&= 0.188 \pm 0.002 B_2^g \pm 0.019(\mathrm{param.}) \pm 0.009(\mathrm{syst.})\,. \qquad (4.2)
\end{aligned}$$

The entry labelled T4 also contains an estimate of the possible impact of the local twist-4 two-gluon matrix elements in (2.19). For this estimate, we exploit the fact that the asymptotic DA of the non-local generalisation of (2.19) is the same as for the twist-2 two-quark DA: $6u(1-u)$.[4] We then assume that the corresponding correlation function is the same as that for the leading conformal wave in the two-quark twist-2 contribution, i.e. the coefficient in the Gegenbauer moment $a_0 = 1$, and replace a_0 by $\langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta_{q,s}\rangle/(f_{q,s}m_b^2)$. The factor $1/m_b^2$ comes from the fact that this is a twist-4 effect and hence suppressed by two powers of m_b with respect to the twist-2 contribution. This is only a rough estimate, of course, as the true spectral density will be different. The result in (4.2) shows that for small $B_2^g \approx 2$ both twist-2 and -4 two-gluon effects can indeed be of similar size. In this case, however, the total flavour singlet contribution to $f_+^{\eta'}$ will also be small, ~ 0.008. In the third lines, we have added all uncertainties from the input parameters (param.) in quadrature and the sum-rule specific uncertainties from M^2 and s_0 (syst.) linearly. For $f_+^{\eta'}(0)$, we have displayed the dependence on B_2^g separately. Our new result for $f_+^\eta(0)$ is, within errors, in agreement with our previous one, $f_+^\eta(0) = 0.275 \pm 0.036$, obtained in ref. [25]. That for $f_+^{\eta'}(0)$ is new. Our results agree well with those obtained in ref. [22], from perturbative QCD factorisation, $f_+^\eta(0) = 0.208$ and $f_+^{\eta'}(0) = 0.171$, including a rescaling by a factor $\sqrt{2}$ to bring their definition of the form factors into agreement with ours. We confirm the finding of ref. [22] that the range of the singlet contribution to the form factor estimated in ref. [13] is likely to be too large, unless B_2^g assumes extreme values ~ 40.

In figure 5 we plot the ratio $f_+^{\eta'}(0)/f_+^\eta(0)$ as a function of the Borel parameter. In the ratio, many uncertainties cancel, in particular that on f_B. As we have chosen $B_2^g = 0$ as central value, $f_+^{\eta'}(0)/f_+^{\eta'}(0) \equiv \tan\phi = 0.814$ exactly, see eq. (3.7). The figure also illustrates the change of the result upon inclusion of a non-zero B_2^g (red/dashed-dotted curves). The ratio is actually rather sensitive to that parameter. While the dependence on a_2 largely cancels when a_2^η and $a_2^{\eta'}$ are set equal, there is a considerable residual dependence on $a_2^\eta - a_2^{\eta'} \neq 0$ (green/short-dashed curves). While $|a_2^\eta - a_2^{\eta'}| = 0.3$ as illustrated by these curves is rather unlikely, and would signal very large OZI-breaking contributions (recall that $a_2^\eta \neq a_2^{\eta'}$ or, equivalently, $a_2^1 \neq a_2^8$ signals the presence of "wrong-flavour" contributions to the $\eta_{q,s}$ DAs and is set to 0 in the QF mixing scheme), one should nonetheless keep in mind

[4]This follows from the general formula for asymptotic DAs, $u^{2j_1-1}(1-u)^{2j_2-1}$, with $j = 1/2(l+s)$ the lowest conformal spin of the operator, and l its canonical dimension, s the Lorentz-spin projection. For $G_{\perp\perp}$, one has $l = 2$ and $s = 0$ [29].

that moderate corrections of this type are not excluded and compete with the OZI-allowed corrections in B_2^g.

Let us now turn to the dependence of the form factors on q^2. In figure 6 we show this dependence in the range $0 < q^2 < 16\,\text{GeV}^2$ accessible by LCSRs. Again we display in blue (by long-dashed curves) the dependence of $f_+^{\eta^{(\prime)}}(q^2)$ on the sum-rule specific parameters M^2 and s_0, the green (short-dashed) curves illustrate the dependence on a_2 and other parameters and the red (dash-dotted) ones that on B_2^g. We give two different parametrisations of the form factors, in terms of a sum of two poles, the so-called BZ parametrisation as given in ref. [25], and in terms of the BGL parametrisation based on analyticity of f_+ in q^2 [37]. Both parametrisations are fitted to the LCSR results in the range $0 < q^2 < 16\,\text{GeV}^2$, and can then be used to extrapolate these results to $q^2_{\text{max}} = (m_B - m_{\eta^{(\prime)}})^2$; this is possible as both parametrisations include the essential feature of the $B^*(1^-)$ pole at $q^2 = m_{B^*}^2$, $m_{B^*} = 5.33\,\text{GeV}$, which governs the large-q^2 behaviour of $b \to u$ vector-current transitions close to q^2_{max}.

The BZ parametrisation reads

$$f_+(q^2) = f_+(0)\left(\frac{1}{1-q^2/m_{B^*}^2} + \frac{rq^2/m_{B^*}^2}{\left(1-q^2/m_{B^*}^2\right)\left(1-\alpha\, q^2/m_B^2\right)}\right), \tag{4.3}$$

with the two shape parameters α, r and the normalisation $f_+(0)$. The BGL parametrisation, on the other hand, is given by

$$f_+(q^2) = \frac{1}{P(q^2)\phi(q^2,q_0^2)} \sum_{k=0}^{\infty} a_k(q_0^2)[z(q^2,q_0^2)]^k\,, \tag{4.4}$$

$$\begin{aligned}
\text{with}\quad z(q^2,q_0^2) &= \frac{\{q_+^2-q^2\}^{1/2}-\{q_+^2-q_0^2\}^{1/2}}{\{q_+^2-q^2\}^{1/2}+\{q_+^2-q_0^2\}^{1/2}}\,,\\
\phi(q^2,q_0^2) &= \frac{(q_+^2-q^2)(\sqrt{q_+^2-q_-^2}+\sqrt{q_+^2-q^2})^{3/2}(\sqrt{q_+^2-q^2}+\sqrt{q_+^2-q_0^2})}{(\sqrt{q_+^2}+\sqrt{q_+^2-q^2})^5(q_+^2-q_0^2)^{1/4}}\,,\\
\text{and}\quad q_\pm^2 &= (m_B \pm m_{\eta^{(\prime)}})^2\,.
\end{aligned} \tag{4.5}$$

The "Blaschke" factor $P(q^2) = z(q^2, m_{B^*}^2)$ accounts for the B^* pole. q_0^2 is a free parameter that can be chosen to attain the tightest possible bounds, and it defines $z(q_0^2,q_0^2) = 0$. One has $|z| < 1$ for $q_0^2 < (m_B + m_{\eta^{(\prime)}})^2$. In the following we choose q_0^2 such that $z(0,q_0^2) \equiv -z(q_-^2,q_0^2)$, i.e. $q_0^2 = 14.14\,\text{GeV}^2$ for η and $10.85\,\text{GeV}^2$ for η'. With these values, $|z|$ becomes minimal: $|z| < 0.13$ for η and $|z| < 0.08$ for η'. The series in (4.4) provides a systematic expansion in the small parameter z, which for practical purposes has to be truncated at order k_{max}. In this paper, we choose $k_{\text{max}} = 3$.

The advantage of the BZ parametrisation is that it is both intuitive and simple: it can be obtained from the dispersion relation for f_+,

$$f_+^{\eta^{(\prime)}}(q^2) = \frac{\text{Res}_{q^2=m_{B^*}^2} f_+(q^2)}{q^2-m_{B^*}^2} + \frac{1}{\pi}\int_{(m_B+m_{\eta^{(\prime)}})^2}^{\infty} dt\, \frac{\text{Im}\, f_+^{\eta^{(\prime)}}(t)}{t-q^2-i\epsilon}\,, \tag{4.6}$$

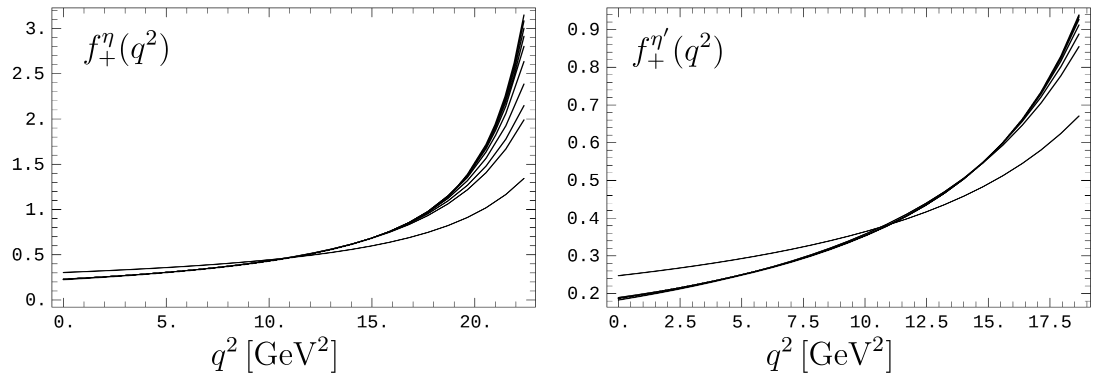

Figure 7: $f_+^{\eta^{(\prime)}}(q^2)$ for central values of input parameters, fitted to the BGL parametrisation (4.4), for $0 \leq k_{\max} \leq 9$.

by replacing the second term on the right-hand side by an effective pole. However, it cannot easily be extended to include more parameters. The strength of the BGL parametrisation, on the other hand, is that the dominant behaviour in q^2 close to the pole at $m_{B^*}^2$ is factored out and the remaining q^2-dependence is organised as a Taylor-series in the small q^2-dependent parameter z; the truncation of the series can be adjusted to the accuracy of the available input parameters. In figure 7 we plot $f_+^{\eta^{(\prime)}}(q^2)$ parametrised à la BGL for $0 \leq k_{\max} \leq 9$. Obviously, the parametrisations converge rapidly with increasing $k_{\max}$ and only differ at very large q^2. The impact of this difference on the predicted branching ratio (3.3) is however only minor, as this region is phase-space suppressed. In the following, we choose $k_{\max} = 3$, which ensures that the total predicted branching ratio agrees within 1% with that obtained for $k_{\max} = 9$.

In table 1 we give the best-fit parameters for $f_+^{\eta^{(\prime)}}$ in the BZ parametrisation, with the small effects of non-zero B_2^g expanded linearly in that parameter. Table 2 contains the corresponding parameters for the BGL parametrisation with $k_{\max} = 3$. Finally, in figure 8 we show the dependence of the ratio of branching ratios $R_{\eta\eta'} = \mathcal{B}(B \to \eta' e\nu)/\mathcal{B}(B \to \eta e\nu)$ on B_2^g. The advantage of this observable is that all hadronic effects are encoded in the form factors and that $|V_{ub}|$ cancels. The blue (solid) curve corresponds to the branching ratios obtained from the central values of input parameters; the dependence of these predictions on the cut-off in k is very small: the long-dashed (blue) curves illustrate the dependence on $k_{\max} = 3 \pm 1$. On the other hand, $R_{\eta\eta'}$ also depends on $a_2^\eta \neq a_2^{\eta'}$. This dependence is shown by the red (short-dashed) curves. The conclusion is that large values of B_2^g, $|B_2^g| > 5$, can be distinguished from the OZI-breaking parameter $|a_2^\eta - a_2^{\eta'}|$, once an accurate experimental value of $R_{\eta\eta'}$ is available, but that for smallish B_2^g and unknown $|a_2^\eta - a_2^{\eta'}|$ only mutual constraints on these parameters can be extracted from the data. In this case, as mentioned before, also twist-4 gluonic DAs can become important.

To summarise, we have calculated the form factors of $B \to \eta^{(\prime)}$ semileptonic transitions from QCD sum rules on the light cone, including the gluonic singlet contributions. We have found that, as expected, these contributions are more relevant for $f_+^{\eta'}$ than for f_+^η and can amount up to 20% in the former, depending on the only poorly constrained leading Gegen-

	$f_+(0)$	α	r
η	$0.231^{+0.018}_{-0.020}$	$0.851^{+0.183}_{-0.492}$	$0.411^{-0.030}_{+0.119}$
η'	$0.189^{+0.015}_{-0.016} + B_2^g \binom{+0.002}{-0.002}$	$0.851^{+0.185}_{-0.497} + B_2^g \binom{-0.006}{+0.008}$	$0.411^{-0.031}_{+0.122} + B_2^g \binom{+0.005}{-0.006}$

Table 1: Parameters for the BZ parametrisation (4.3). The uncertainty contains all sources of error added in quadrature, except for η', where the uncertainty in B_2^g is approximated by a linear term. The upper (lower) terms represent the maximum (minimum) value of the form factor.

	η	η'
a_0	0.0031 ± 0.0003	$0.0018 \pm 0.0002 \pm 0.00002 B_2^g$
a_1	-0.0090 ∓ 0.0034	$-0.0058 \mp 0.0016 \mp 0.0001 B_2^g$
a_2	0.0243 ± 0.0172	$0.0174 \pm 0.0166 \mp 0.0001 B_2^g$
a_3	-0.0908 ∓ 0.0039	$-0.1189 \mp 0.0218 \pm 0.0016 B_2^g$

Table 2: Like tabular 1, but for the BGL parametrisation (4.4) with $k_{\max} = 3$.

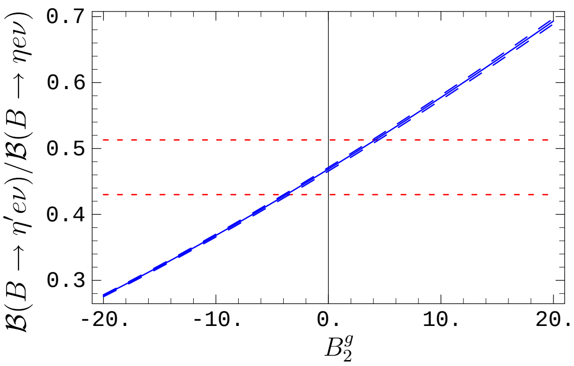

Figure 8: [Colour online] The ratio of branching ratios $R_{\eta\eta'} = \mathcal{B}(B \to \eta' e\nu)/\mathcal{B}(B \to \eta e\nu)$ as a function of the singlet-parameter B_2^g. Solid (blue) curve: central values of input parameters and BGL parametrisation with $k_{\max} = 3$; long-dashed (blue) curves: BGL parametrisations with $k_{\max}$ varied by ± 1. Short-dashed (red) curves: theoretical uncertainty of $R_{\eta\eta'}$ for $B_2^g = 0$, for $a_2^{\eta,\eta'}(1\,\mathrm{GeV})$ varied independently, as in figure 5.

bauer moment B_2^g of the gluonic twist-2 distribution amplitude of $\eta^{(\prime)}$. We also found that the form factors are sensitive to the values of the twist-2 two-quark Gegenbauer moments $a_2^{\eta,\eta'}$ which, given the uncertainty of independent determinations, we have set equal to a_2^π. The ratio of branching ratios $\mathcal{B}(B \to \eta' e\nu)/\mathcal{B}(B \to \eta e\nu)$ is sensitive to both a_2 and B_2^g and may be used to constrain these parameters, once it is measured with sufficient accuracy. The extraction of $|V_{ub}|$ from these semileptonic decays, in particular $B \to \eta e\nu$, with negligible singlet contribution, although possible in principle, at the moment is obscured by the lack of knowledge of a_2. We would also like to stress that, in the framework of the quark-flavour mixing scheme for the η-η' system as used in this paper, $B \to \eta^{(\prime)}$ transitions probe only the η_q component of these particles. The η_s component could be probed directly for instance in the $b \to s$ penguin transition $B_s \to \eta^{(\prime)}\ell^+\ell^-$, although such a measurement would also be sensitive to new physics in the penguin diagrams.

Acknowledgments

G.W.J. gratefully acknowledges receipt of a UK PPARC studentship. This work was supported in part by the EU networks contract Nos. MRTN-CT-2006-035482, Flavianet, and MRTN-CT-2006-035505, Heptools.

A. Spectral density of the two-gluon contribution to f_+

The contribution of the twist-2 two-gluon distribution amplitude to the correlation functions Π_+^η and $\Pi_+^{\eta'}$, eq. (3.7), is given by

$$\Pi_+^{P,1} = \int_{m_b^2}^{\infty} ds\, \frac{\rho_1^P(s)}{s - p_B^2}$$

with

$$\begin{aligned} \rho_1^P(s) = {} & B_2^g a_s f_1^P m_b \frac{5}{36\sqrt{3}} \frac{m_b^2 - s}{(s-q^2)^5} \left\{59 m_b^6 + 21 q^6 - 63 q^4 s - 19 q^2 s^2 + 2 s^3 \right. \\ & \left. + m_b^2 s (164 q^2 + 13 s) - m_b^4 (82 q^2 + 95 s)\right\} \\ & + B_2^g a_s f_1^P m_b \frac{5}{6\sqrt{3}} \frac{(m_b^2 - q^2)(s - m_b^2)}{(s-q^2)^5} \left\{5 m_b^4 + q^4 + 3 q^2 s + s^2 - 5 m_b^2 (q^2 + s)\right\} \\ & \times \left\{2 \ln \frac{s - m_b^2}{m_b^2} - \ln \frac{\mu^2}{m_b^2}\right\}. \end{aligned} \tag{A.1}$$

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